

15.8 Videos Guide

15.8a

- Rectangular-spherical conversions
 - $x = \rho \cos \theta \sin \phi$
 - $y = \rho \sin \theta \sin \phi$
 - $z = \rho \cos \phi$
 - $x^2 + y^2 + z^2 = \rho^2$

15.8b

Exercise:

- Sketch the solid described by the given inequalities.
 $1 \leq \rho \leq 2, \quad \pi/2 \leq \phi \leq \pi$
- The triple integral in spherical coordinates
 - $\iiint_E f(x, y, z) dV$
 $= \int_c^d \int_\alpha^\beta \int_{g_1(\theta, \phi)}^{g_2(\theta, \phi)} f(\rho \cos \theta \sin \phi, \rho \sin \theta \sin \phi, \rho \cos \phi) \rho^2 \sin \phi d\rho d\theta d\phi$
 - Note that $dV = \rho^2 \sin \phi d\rho d\theta d\phi$

Exercises:

15.8c

- Sketch the solid whose volume is given by the integral and evaluate the integral.

$$\int_0^{\pi/4} \int_0^{2\pi} \int_0^{\sec \phi} \rho^2 \sin \phi d\rho d\theta d\phi$$

15.8d

- Use spherical coordinates to evaluate $\iiint_E y^2 x^2 dV$, where E lies above the cone $\phi = \pi/3$ and below the sphere $\rho = 1$.

15.8e

- Evaluate the integral by changing to spherical coordinates.

$$\int_{-a}^a \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} \int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} (x^2 z + y^2 z + z^3) dz dx dy$$